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D Nicolaides

Natural Sciences and Mathematics, Bloomfield College, 467 Franklin Street, Bloomfield, NJ 07003, USA

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Abstract. An exact model which partially takes into account interactions of fluctuations is used to study the critical behaviour at phase transitions of coupled systems with cubic interactions. On the basis of the relationship between the coupling potentials, it is shown that the fluctuation-induced first-order phase transition which occurs in uncoupled cubic anisotropic systems may now be replaced by a new second-order one.

Since the introduction of renormalization group (RG) theory in the 1970s, many problems in the theory of phase transitions that had resisted other approaches have been successfully considered and many results have been experimentally confirmed [1–4]. Even though RG theory is the most appropriate theory for studying critical phenomena, frequently this approach does not give explicit results, especially for systems in three dimensions. The main reason for this is the complexity of the symmetry of the free-energy functional that describes realistic systems. Examples include systems with or without the influence of quenched disorder described by a single or multiple coupled order parameters, systems with cubic or dipole interactions, and systems with short- or long-range-correlation impurities either in the static or dynamic approach. Using exact models that partially take into account interactions of fluctuations [5–11], some of these systems were successfully treated theoretically. This provides not only an alternative approach for the study of complex-symmetry systems, but also a check on the validity of RG predictions which are very often based on various approximations such as the ε-expansion.

In this paper the critical behaviour of a system with two coupled vector order parameters and cubic anisotropy is studied using an exact model which takes into account fluctuation interactions of equal and oppositely directed momenta. The isotropic two-coupled-parameter system, with or without the presence of two random fields, was studied previously by means of both the RG theory and the exact model. Without the random fields, RG theory [2, 3, 12, 13] predicts a fluctuation-induced first-order transition which is rigorously borne out [8] by the model. The effect of the random fields has not yet been considered in RG theory, but the model explicitly shows the second-order phase transition to be restored in the presence of one random field and that the ordered phase for space dimensionality $d \leqslant 4$ is destroyed when both random fields are present [10]. The addition of a cubic anisotropy term into the free-energy functional of a pure coupled-parameter system changes its critical behaviour and this is the subject here. In particular, it is shown that the anisotropic coupled-parameter system exhibits a new second-order phase transition which replaces the discontinuous transition present in an uncoupled cubic anisotropic system. Also, a new fluctuation-induced first-order transition in the anisotropic phase occurs.

To show the important steps for the exactly solvable model, we start by considering the isotropic $\varphi^4(\boldsymbol{x})$ model with the Ginzburg–Landau functional having a scalar order parameter $\varphi(\boldsymbol{x})$:

$$F[\varphi(\boldsymbol{x})] = \frac{1}{2}\int \mathrm{d}^d x \left[\tau\varphi^2(\boldsymbol{x}) + c(\nabla\varphi(\boldsymbol{x}))^2 + \frac{1}{4}g\varphi^4(\boldsymbol{x}) - h\varphi(\boldsymbol{x})\right] \tag{1}$$

where $\tau = (T - T_c)/T_c$, T_c is a trial critical temperature, and h is a constant field. The partition function is obtained by adding up the contributions for all possible functions $\varphi(\boldsymbol{x})$ in the functional integral

$$Z = \int \mathrm{D}\varphi(\boldsymbol{x}) \exp(-F[\varphi(\boldsymbol{x})]).$$

Due to the existence of the interaction term in functional (1), exact evaluation of the partition function is impossible. However, such a difficulty is eliminated when the interaction term is reduced according to the following:

$$\int \mathrm{d}^d x\ \varphi^4(\boldsymbol{x}) \to \frac{1}{V}\left[\int \mathrm{d}^d x\ \varphi^2(\boldsymbol{x})\right]^2 \equiv \frac{a^2[\varphi(\boldsymbol{x})]}{V} \tag{2}$$

where V is the volume of the system. This reduction was first proposed in reference [14]. In momentum space, the initial δ-function, $\delta(\boldsymbol{p}_1 + \boldsymbol{p}_2 + \boldsymbol{p}_3 + \boldsymbol{p}_4)$, which is responsible for the momentum conservation, splits into a product of two δ-functions, $\delta(\boldsymbol{p}_1 + \boldsymbol{p}_2)\delta(\boldsymbol{p}_3 + \boldsymbol{p}_4)$. The physical meaning of this is that, while preserving the symmetry, the model takes into consideration fluctuation interactions of equal and antiparallel momenta. This reduction transfers the $\varphi^4(\boldsymbol{x})$ model into the universality class of the spherical model [14, 15].

In order to calculate functional integrals with respect to $\varphi(\boldsymbol{x})$ one has to transform the $\varphi^4(\boldsymbol{x})$ term into a bilinear form. So far, functional (1) has been considered in the form

$$F[\varphi(\boldsymbol{x})] = \frac{\tau}{2}a[\varphi(\boldsymbol{x})] + \frac{1}{8V}ga^2[\varphi(\boldsymbol{x})] + \frac{1}{2}\int \mathrm{d}^d x\ \left[c(\nabla\varphi(\boldsymbol{x}))^2 - h\varphi(\boldsymbol{x})\right].$$

To cast it in a bilinear form with respect to $\varphi(\boldsymbol{x})$, we use a transformation analogous to that of Hubbard and Stratonovich:

$$\exp\left[-\frac{V}{2}K\left(\frac{a[\varphi]}{V}\right)\right] = \frac{1}{(2\pi)}\int \mathrm{d}x\ \mathrm{d}y\ \exp\left[-\frac{V}{2}K\left(\frac{x}{V}\right) + \mathrm{i}(xy - ya[\varphi])\right] \tag{3}$$

where $K(x/V)$ is an arbitrary function. In the case of functional (1), $K(x/V)$ is

$$K\left(\frac{x}{V}\right) = \frac{\tau}{V}x + \frac{g}{4V^2}x^2.$$

Even though, on the one hand, this simplifies the functional integration, on the other hand, two more variables are added in the equations, x and y. However, this can be handled. After applying the transformation (3), Gaussian integrals with respect to $\varphi(\boldsymbol{x})$ in the partition function can be calculated. As a result, the partition function takes the form

$$Z \propto \int_{-\infty}^{\infty} \mathrm{d}x\ \mathrm{d}y \exp\left[-\frac{V}{2}\left(\tau x + \frac{g}{4}x^2 - xy + \frac{1}{V}\sum_{\boldsymbol{q}} \ln|c\boldsymbol{q}^2 + y| - \frac{h^2}{4y}\right)\right].$$

Summations of the kind $\sum_{\boldsymbol{q}} \ln|c\boldsymbol{q}^2 + y|$ have to be cut off since they diverge in the upper limit. However, the critical asymptotics should not depend upon the momentum cut-off. For the spatial dimension in the range $2 < d < 4$, this can be handled by renormalizing the summation and then setting the momentum cut-off to be equal to infinity. For $d \geqslant 4$ the sum becomes non-renormalizable and we must maintain the momentum cut-off explicitly. As we demonstrate below, the dependence upon the momentum cut-off is absorbed into a renormalization of the trial value of the critical temperature and into an insignificant constant addition to the free energy. If the cut-off momentum is equal to Λ, then

$$\sum_{\boldsymbol{q}} \ln|c\boldsymbol{q}^2 + y| = V\,[f_d(y) + y\Theta_d(\Lambda)]$$

where

$$f_d(y) = \frac{S_d}{(2\pi)^d}\begin{cases} \dfrac{\pi y^{d/2}}{d c^{d/2}\sin(\pi d/2)} \equiv \kappa(c) y^{d/2} & d \neq \text{even} \\ -\dfrac{1}{d}\left(\dfrac{y}{c}\right)^{d/2} \ln y \equiv \mu(c) y^{d/2} \ln y & d = \text{even} \end{cases}$$

$$\Theta_d(\Lambda) = \frac{S_d}{(2\pi)^d}\begin{cases} \dfrac{\Lambda^{(d-2)}}{c(d-2)} & d \neq 2 \\ \dfrac{\ln(c\Lambda^2)}{2c} & d = 2. \end{cases}$$

S_d is the surface area of a d-dimensional unit-radius sphere. $\Theta_d(\Lambda)$ is used to renormalize $x \to x + \Theta_d(\Lambda)$, which consequently results in the renormalization of the trial critical temperature τ, $2t = 2\tau + g\Theta_d(\Lambda)$. As a result, the partition function becomes

$$Z \propto \int_{-\infty}^{\infty} \mathrm{d}x\ \mathrm{d}y \exp\left\{-\frac{V}{2} F(x, y, h)\right\} \tag{4}$$

with the non-equilibrium free energy $F(x, y, h)$ given by

$$F(x, y, h) = x(t - y) + \frac{g}{4}x^2 + f_d(y; c) - \frac{h^2}{y}. \tag{5}$$

In the thermodynamic limit, $V \to \infty$, the integrals in (4) can be calculated exactly using the steepest-descent method. Hence, all of the thermodynamic quantities can be calculated. The free-energy density is given by equation (5) with x and y being solutions of the system of equations $\partial F/\partial x = 0$, $\partial F/\partial y = 0$. After eliminating x from these equations, we derive an equation for y:

$$0 = t - y + \frac{g}{2}\left(\frac{3}{2}\kappa(c) y^{1/2} + \frac{h^2}{y^2}\right).$$

An equilibrium value of the order parameter at zero external field is given by

$$\varphi = -\lim_{h\to 0} \frac{\partial F}{\partial h} = \lim_{h\to 0} \frac{h}{y}.$$

Now, let us apply the above steps in the case of the Ginzburg–Landau–Wilson functional of two coupled, m-component systems with cubic anisotropy. The functional of interest is of the form

$$\begin{aligned} F(\boldsymbol{\varphi}_1, \boldsymbol{\varphi}_2) = \frac{1}{2}\int \mathrm{d}^d x \,\Bigg\{\sum_{i=1}^{2}\Bigg[\tau_i|\boldsymbol{\varphi}_i(\boldsymbol{x})|^2 + c_i(\nabla\boldsymbol{\varphi}_i(\boldsymbol{x}))^2 + \frac{1}{4}g_i|\boldsymbol{\varphi}_i(\boldsymbol{x})|^4 \\ + v_i\sum_{\alpha=1}^{m}\varphi_{i\alpha}^4(\boldsymbol{x}) - \boldsymbol{h}_i\cdot\boldsymbol{\varphi}(\boldsymbol{x})\Bigg] + \frac{1}{2}w|\boldsymbol{\varphi}_1(\boldsymbol{x})|^2|\boldsymbol{\varphi}_2(\boldsymbol{x})|^2\Bigg\} \end{aligned} \tag{6}$$

where $\tau_i = (T - T_{ci})/T_{ci}$, and T_{ci} is a trial critical temperature for the m-component order parameter $\boldsymbol{\varphi}_i(\boldsymbol{x})$, and $\boldsymbol{h}_i$ is a constant external conjugate m-component field.

The partition function is obtained by adding up the contributions for all possible functions $\boldsymbol{\varphi}_i(\boldsymbol{x})$ in the functional integral

$$Z = \int \mathrm{D}\boldsymbol{\varphi}_1\,\mathrm{D}\boldsymbol{\varphi}_2 \exp(-F(\boldsymbol{\varphi}_1, \boldsymbol{\varphi}_2)).$$

The exact evaluation of the partition function will be possible when all of the interaction terms in functional (6) are reduced according to (2). Specifically for the coupled cubic anisotropic system, we have

$$\int \mathrm{d}^d x\ \varphi_{i\alpha}^4(\boldsymbol{x}) \rightarrow \frac{1}{V}\left[\int \mathrm{d}^d x\ \varphi_{i\alpha}^2(\boldsymbol{x})\right]^2 \equiv \frac{a_{i\alpha}^2[\varphi_{i\alpha}(\boldsymbol{x})]}{V}. \tag{7}$$

This reduction is one of the basic characteristics of the exact model. When the model was used for cubic anisotropic systems, $w = 0$ in functional (6), (i) the anisotropic phase having only one non-zero component of the vector order parameter along the side of the cube and (ii) the isotropic phase having the vector order parameter along the diagonal of the cube with all components equal to one another could occur under different circumstances [7]. Explicitly for $d = 3$, when

$$v_i > 0 \tag{8}$$

or when simultaneously the inequalities

$$\begin{aligned} &v_i < 0\\ &4v_i + mg_i > 0\\ &4v_i + g_i < 0 \end{aligned} \tag{9}$$

are satisfied, there exists a second-order phase transition into the isotropic phase. On the other hand, when

$$\begin{aligned} &v_i < 0\\ &4v_i + g_i > 0 \end{aligned} \tag{10}$$

simultaneously hold, the transition is of first order into the anisotropic phase. If however the inequalities

$$\begin{aligned} &v_i < 0\\ &4v_i + mg_i < 0 \end{aligned} \tag{11}$$

hold, the solution for the equilibrium order parameter corresponds to an unbounded behaviour, which means that the system can never have long-range order.

When $v_i = 0$ and $w \neq 0$, functional (6) represents a system with two coupled order parameters without cubic interactions. Studying such systems within the context of the exact model, it is shown explicitly that a fluctuation-induced first-order phase transition into the isotropic phase occurs [8]. Mean-field theory finds that a disorder–order phase transition is always of second order. The model is in agreement with RG theory predictions [2, 3, 12, 13].

It will be shown below that for functional (6) the coupling will induce new continuous phase transitions which are w-dependent. In addition, a new first-order transition will still occur but the system will always be anisotropically ordered, indicating the domination of the cubic anisotropy interaction over the effect of coupling between the systems.

After reduction (7), the exponent in the partition function becomes a quadratic form with respect to the functionals $a_{i\alpha}[\varphi_{i\alpha}]$. Then a transformation analogous to that of Hubbard and Stratonovich is used:

$$\exp\left(-\frac{V}{2}K\left(\frac{a_{i\alpha}[\varphi_{i\alpha}]}{V}\right)\right) = \frac{1}{(2\pi)^{i\alpha}}\int \mathrm{D}x_{i\alpha}\ \mathrm{D}y_{i\alpha}\ \exp\left(-\frac{V}{2}K\left(\frac{x_{i\alpha}}{V}\right) + \mathrm{i}\sum_{i=1}^{2}\sum_{\alpha=1}^{m}(x_{i\alpha}y_{i\alpha} - y_{i\alpha}a_{i\alpha})\right)$$

where $K(x_{i\alpha}/V)$ is an arbitrary function, and, for the case of functional (6), $K(x_{i\alpha}/V)$ is

$$K\left(\frac{x_{i\alpha}}{V}\right) = \sum_{i=1}^{2}\left[\frac{\tau_i}{V}\sum_{\alpha=1}^{m}x_{i\alpha} + \frac{g_i}{4V^2}\left(\sum_{\alpha=1}^{m}x_{i\alpha}\right)^2 + \frac{v_i}{V^2}\sum_{\alpha=1}^{m}x_{i\alpha}^2\right] + \frac{w}{2V^2}\sum_{\alpha=1}^{m}x_{1\alpha}\sum_{\alpha=1}^{m}x_{2\alpha}.$$

The partition function in momentum space then becomes

$$Z \propto \int \mathrm{D}\varphi_{1\alpha\boldsymbol{q}}\,\mathrm{D}\varphi_{2\alpha\boldsymbol{q}}\int_{-\infty}^{\infty}\mathrm{D}x_{1\alpha}\,\mathrm{D}x_{2\alpha}\int_{-\mathrm{i}\infty}^{\mathrm{i}\infty}\mathrm{D}y_{1\alpha}\,\mathrm{D}y_{2\alpha}\exp\left\{-\frac{V}{2}\left(K\left(\frac{x_{i\alpha}}{V}\right)\right.\right.$$
$$\left.\left.-\frac{1}{V}\sum_{i=1}^{2}\sum_{\alpha=1}^{m}(2\mathrm{i}x_{i\alpha}y_{i\alpha} + h_{i\alpha}\varphi_{i\alpha0}\right) + \frac{1}{V^2}\sum_{i=1}^{2}\sum_{\alpha=1}^{m}\sum_{\boldsymbol{q}}(2iy_{i\alpha} + c_i\boldsymbol{q}^2)|\varphi_{i\alpha\boldsymbol{q}}|^2\right)\right\}. \quad (12)$$

After defining $x_{i\alpha}/V \to x_{i\alpha}$, $2\mathrm{i}y_{i\alpha} \to y_{i\alpha}$, $\varphi_{i\alpha\boldsymbol{q}}/\sqrt{V} \to \varphi_{i\alpha\boldsymbol{q}}$, integrations over all possible modes $\varphi_{i\alpha\boldsymbol{q}}$ may be performed to obtain

$$Z \propto \int_{-\infty}^{\infty}\mathrm{D}x_{1\alpha}\,\mathrm{D}x_{2\alpha}\int_{-\infty}^{\infty}\mathrm{D}y_{1\alpha}\,\mathrm{D}y_{2\alpha}\;\exp\left\{-\frac{V}{2}\left(K(x_{i\alpha})\right.\right.$$
$$\left.\left.-\sum_{i=1}^{2}\sum_{\alpha=1}^{m}\left(x_{i\alpha}y_{i\alpha} + \frac{h_{i\alpha}^2}{4Vy_{i\alpha}} - \frac{1}{V}\sum_{\boldsymbol{q}}\ln|c_i\boldsymbol{q}^2 + y_{i\alpha}|\right)\right)\right\}.$$

Treating the summation $\sum_{\boldsymbol{q}}\ln|c_i\boldsymbol{q}^2 + y_{i\alpha}|$ as in the simple φ^4-case, and for $d = 3$, one finds

$$\sum_{\boldsymbol{q}}\ln|c_i\boldsymbol{q}^2 + y_{i\alpha}| = V(f(y_{i\alpha};c_i) + y_{i\alpha}\Theta(\Lambda;c_i))$$

where

$$\Theta(\Lambda;c_i) = \frac{S_3}{(2\pi)^3}\frac{\Lambda}{c_i}$$
$$f(y_{i\alpha};c_i) = \frac{-S_3}{(2\pi)^3}\frac{\pi y_{i\alpha}^{3/2}}{3c_i^{3/2}} \equiv \kappa(c_i)y_{i\alpha}^{3/2}.$$

$\Theta(\Lambda;c_i)$ is used to renormalize $x_{i\alpha}$; that is,

$$x_{i\alpha} \to x_{i\alpha} + \Theta(\Lambda;c_i)$$

which consequently results in the renormalization of the trial critical temperature τ_i, that is,

$$2t_i \equiv 2\tau_i + g_i m\Theta(\Lambda;c_i) + 4\Theta(\Lambda;c_i)v_i + wm\Theta(\Lambda;c_{i'}) \equiv 2(T - T_i)/T_i.$$

After defining $h_{i\alpha}/2\sqrt{V} \to h_{i\alpha}$, the partition function becomes

$$Z \propto \int_{-\infty}^{\infty}\mathrm{D}x_{1\alpha}\,\mathrm{D}x_{2\alpha}\int_{-\infty}^{\infty}\mathrm{D}y_{1\alpha}\,\mathrm{D}y_{2\alpha}\;\exp\left\{-\frac{V}{2}F(x_{i\alpha},y_{i\alpha},h_{i\alpha})\right\} \quad (13)$$

with the non-equilibrium free energy $F(x_{i\alpha},y_{i\alpha},h_{i\alpha})$ given by

$$F(x_{i\alpha},y_{i\alpha},h_{i\alpha}) = \sum_{i=1}^{2}\sum_{\alpha=1}^{m}\left(t_i x_{i\alpha} + v_i x_{i\alpha}^2 - x_{i\alpha}y_{i\alpha} + f(y_{i\alpha};c_i) - \frac{h_{i\alpha}^2}{y_{i\alpha}}\right) + \sum_{i=1}^{2}\frac{g_i}{4}\left(\sum_{\alpha=1}^{m}x_{i\alpha}\right)^2$$
$$+ \frac{w}{2}\sum_{\alpha=1}^{m}x_{1\alpha}\sum_{\alpha=1}^{m}x_{2\alpha}. \quad (14)$$

In the thermodynamic limit, $V \to \infty$, one can calculate the integrals in equation (14) exactly using the method of steepest descent. The equilibrium free-energy density is given by equation (14) with $x_{i\alpha}$ and $y_{i\alpha}$ being the solutions of the system of equations $\partial F/\partial x_{i\alpha} = 0$, $\partial F/\partial y_{i\alpha} = 0$. After eliminating $x_{i\alpha}$, an equation for $y_{i\alpha}$ is derived:

$$0 = t_i + 2v_i\left(\frac{3}{2}\kappa(c_i)y_{i\alpha}^{1/2} + \frac{h_{i\alpha}^2}{y_{i\alpha}^2}\right) - y_{i\alpha} + \frac{g_i}{2}\sum_{\alpha=1}^{m}\left(\frac{3}{2}\kappa(c_i)y_{i\alpha}^{1/2} + \frac{h_{i\alpha}^2}{y_{i\alpha}^2}\right) + \frac{w}{2}\sum_{\alpha=1}^{m}\left(\frac{3}{2}\kappa(c_{i'})y_{i'\alpha}^{1/2} + \frac{h_{i'\alpha}^2}{y_{i'\alpha}^2}\right). \tag{15}$$

Finally, at zero external fields, an expression of the order parameter $\varphi_{i\alpha}$ corresponding to the lowest energy of the system is given by

$$\varphi_{i\alpha} = -\lim_{h_{i\alpha}\to 0}\frac{\partial F}{\partial h_{i\alpha}} = \lim_{h_{i\alpha}\to 0}\frac{h_{i\alpha}}{y_{i\alpha}}. \tag{16}$$

The $4m$ equations (15), (16) contain $4m$ unknowns, the $y_{i\alpha}$ and the $\varphi_{i\alpha}$. These $4m$ equations and unknowns can be proven to be reducible to simply four equations and four unknowns. The physical meaning of this is that when a phase transition occurs into the phase described by $\varphi_{i\alpha}$ with $\alpha = \{1, \ldots, m_i\}$ ($m_i \leqslant m$ and $i = 1$ or 2), having the first m_i components of the order parameter non-zero and the rest zero, all non-zero components of the order parameter are equal to one another. Hence $\varphi_{i\alpha} \equiv \varphi_i$ and $y_{i\alpha} \equiv y_i$, and the four equations are

$$\varphi_i^2 = \frac{3}{2}\kappa(c_i)y_i^{1/2} - \frac{y_i}{2v_i}$$

$$t_i + 3\kappa(c_i)y_i^{1/2}\left(v_i + \frac{mg_i}{4}\right) - y_i\left(1 + \frac{g_i m_i}{4v_i}\right) + \frac{3}{4}w\kappa(c_{i'})my_{i'}^{1/2} - \frac{wm_{i'}}{4v_{i'}}y_{i'} = 0$$

where $i = \{1, 2\}$, $i' = \{2, 1\}$. For a small coupling constant, the anisotropic solution $\varphi_{i\pm}$ with $m_i = 1$ and $m_{i'} = 0$ is given by

$$\varphi_{i\pm}^2 = \frac{3}{2}\kappa(c_i)y_{i0\pm}^{1/2} - \frac{y_{i0\pm}}{2v_i} \pm \frac{w(3v_i\kappa(c_i) - 2y_{i0\pm}^{1/2})(3v_{i'}\kappa(c_{i'})my_{i'0\pm}^{1/2})}{6v_i v_{i'}\kappa(c_i)mG_i\sqrt{1 + 16t_i(4v_i + g_i)/[v_i(3\kappa(c_i)mG_i)^2]}} \tag{17}$$

and the isotropic solution $\Phi_{i\pm}$ with $m_i = m$ and $m_{i'} = 0$ is given by

$$\Phi_{i\pm}^2 = -\frac{2t_i}{mG_i} - \frac{(3\kappa(c_{i'}))^2 wmG_{i'}}{16G_i}\left[1 \pm \sqrt{1 + \frac{64t_{i'}}{(3\kappa(c_{i'})mG_{i'})^2}}\right]. \tag{18}$$

In the last two equations, $mG_i = 4v_i + mg_i$, and

$$y_{i0\pm}^{1/2} = \frac{3v_i\kappa(c_i)mG_i}{2(4v_i + g_i)}\left[1 \pm \sqrt{1 + \frac{16t_i(4v_i + g_i)}{v_i(3\kappa(c_i)mG_i)^2}}\right].$$

Solutions corresponding to the phases described by $\varphi_{i'\pm}$ and $\Phi_{i'\pm}$ can be easily obtained when in equations (17), (18) i is interchanged with i'. The mixed isotropic solution with $m_i = m_{i'} = m$ never occurs since it always has a higher free energy than the isotropic solution, equation (18), or the anisotropic one, equation (17). In fact this is true for any mixed phase. Therefore, either the anisotropic phase (17), or the isotropic one, equation (18), may occur under the appropriate conditions.

The most interesting case is that when one uncoupled system, say i, satisfies the condition for a first-order transition, inequalities (10), and the other one, i', satisfies the conditions for a second-order transition, inequalities (8) or (9) with $i \to i'$. When such systems are coupled together, a phase transition will occur in system i, and, depending on the polarity of w, the transition may be a new first-order one, or a restored new second-order one into the anisotropic phase. The existence of the first-order phase transition, which is always into the anisotropic phase, suggests that the cubic anisotropy effect (which for uncoupled systems leads to anisotropic first-order transitions) dominates over the effect of the coupling between the systems (which for rotationally invariant coupled systems leads to isotropic first-order transitions). Explicitly, a positive coupling produces a new fluctuation-induced first-order phase transition into the anisotropic phase with a lower critical temperature than that of the zero-coupling case. This is obtained when systems i and i' simultaneously satisfy inequalities (10), and (8) or (9) with $i \rightarrow i'$, respectively. The solution $\varphi_{i+} = \varphi_{i-}$ of equation (17) gives the critical temperature of the first-order transition into the anisotropic phase, which to order w is

$$t_{ic}(w \neq 0) = t_{ic}(w = 0) - \frac{27 w v_i v_{i'} G_i G_{i'} m^3 \kappa(c_i) \kappa^2(c_{i'})}{16(4v_i + g_i)(4v_{i'} + g_{i'})}$$

where

$$t_{ic}(w = 0) = \frac{-v_i(3\kappa(c_i) m G_i)^2}{16(4v_i + g_i)}. \tag{19}$$

The critical temperature $t_{ic}(w \neq 0)$ is well above the critical temperatures corresponding to the solutions $\varphi_{i\pm}^2 = 0$ and $\Phi_{i\pm}^2 = 0$, indicating that the transition must indeed be of first order. In the zero-coupling limit, the critical temperature simply reduces to the appropriate one of the uncoupled system, equation (19).

On the other hand, the first-order transition of the uncoupled system i is replaced by a new second-order one if $w < 0$. This restoration is a transition into the anisotropic phase, with a critical temperature that decreases with increasing absolute value of the coupling strength w. This result is obtained by numerical analysis. The critical temperature of this transition is the solution of equation (17) when set equal to zero, $\varphi_{i\pm}^2 = 0$, and when the interaction constants, in addition to $w < 0$, also satisfy inequalities (10), for system i, and (8) or (9) with $i \rightarrow i'$, for system i'. This is a cumbersome expression and will not be provided here. An approximate solution cannot be obtained, since in the uncoupled case the existence of the first-order phase transition provides a discontinuity of the order parameter at criticality. However, numerical calculations show that this critical temperature decreases with increasing absolute value of the coupling strength (figure 1). Also, from figure 1 one can see that the slope of the curve becomes steeper with decreasing absolute value of the coupling constant, and as $|w| \rightarrow 0$ it diverges as expected, since in this limit the uncoupled cubic anisotropic

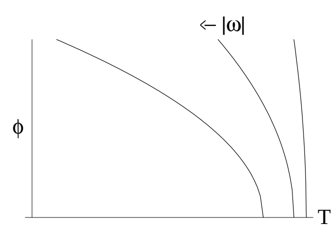

Figure 1. The phase diagram of the equilibrium order parameter of the anisotropic phase φ as a function of temperature for three different strengths of the coupling constant w. The arrow indicates that the direction of increasing $|w|$ is to the left. The critical temperature decreases with increasing $|w|$. Also, the slope of the curve becomes steeper with decreasing $|w|$ and as $|w| \rightarrow 0$ it diverges as expected, since in this limit the uncoupled cubic anisotropic system exhibits a fluctuation-induced first-order phase transition into the anisotropic phase.

system exhibits a fluctuation-induced first-order phase transition into the anisotropic phase. Furthermore, unlike the case for an uncoupled system with cubic anisotropy for which second-order phase transitions are always into the isotropic phase, the new coupling-induced second-order transition may be into the anisotropic phase.

It is also possible for the restoration of the second-order transition, which replaces the first-order one of the uncoupled system, to occur isotropically, a phase described by equation (18). This happens if, before coupling, system i was satisfying the first-order condition (10), and system i' had an unbounded solution, resulting from inequalities (11) with $i \to i'$. After coupling, and for positive w, the critical temperature of the isotropically restored new second-order transition is higher than that for the zero-coupling case, and is given by $\Phi_{i\pm}^2 = 0$, or

$$T_c^{(i\pm)} = T_i - \frac{T_i}{2bw} + \frac{T_i^2}{2bG_{i'}T_{i'}} \pm \frac{1}{2}\sqrt{\left(-2T_i + \frac{T_i}{bw} - \frac{T_i^2}{bG_{i'}T_{i'}}\right)^2 - 4\left(T_i^2 + \frac{T_i^2}{bG_{i'}} - \frac{T_i^2}{bw}\right)}$$

where $b = 16/(9m^2\kappa^2(c_{i'})w^2G_{i'})$. In the zero-coupling limit, it follows from equation (18) that the critical temperature reduces appropriately to $t_i = 0$, which characterizes the second-order transition of the uncoupled system, but at the same time, for $w = 0$, the critical temperature of the first-order anisotropic phase, equation (19), is higher, and therefore prevails over the isotropic phase as expected [7]. On the other hand, for negative w, the system experiences a fluctuation-induced first-order transition into the anisotropic phase of system i as described previously.

As long as system i has a bounded solution satisfying one of the set of inequalities (8), (9), (10), and system i' has an unbounded solution satisfying inequalities (11), when they are coupled together, a phase transition will always occur in system i. This happens regardless of whether system i has a higher or lower trial critical temperature than system i'. Therefore this means that the transition could be an anomalous one, from the mean-field theory point of view, into a phase with a lower trial critical temperature.

Upon suppression of fluctuations by taking the limit $c_i \to \infty$, all of the results reduce to those of mean-field theory as expected.

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